



MATH 038: Elementary Real Analysis

Term: 2020 Winter Session

Instructor: Staff

Language of Instruction: English

Classroom: TBA

Office Hours: TBA

Class Sessions Per Week: 6

Total Weeks: 4

Total Class Sessions: 25

Class Session Length (minutes): 145

Credit Hours: 4

Course Description:

This course introduces students to elementary real analysis, which makes what the students have learned from calculus courses rigorous. The following topics will be covered: the properties of real numbers, sequences, sets of real numbers, continuous functions, the integral, sequences and series of functions, power series, the Euclidean spaces \mathbb{R}^n . In this course, students will learn various facts and practice computing limits and integrals to solve specific problems, which helps students deeply understand and apply the knowledge in calculus class. In addition, instructing students how to read and write proofs is another essential goal of this course.

Prerequisite: MATH 031 Calculus 3

Learning objectives:

Upon successful completion of this course, students will be able to:

- Understand the key concepts in the properties of real numbers
- Understand the meaning of convergence and apply it to sequences, series, and functions
- Solve problems and write proofs of theorems that meet rigorous standards based on content, organization and coherence, argument and support, and style and mechanics
- Determine the Riemann integrability of a bounded function



- Demonstrate the effect of uniform convergence

Course Materials:

1. Brian S. Thomson, Judith B. Bruckner, and Andrew M. Bruckner, *Elementary Real Analysis*, 2nd edition, CreateSpace Independent Publishing Platform, 2008
2. Jiří Lebl, *Basic Analysis: Introduction to Real Analysis*, 4th edition, CreateSpace Independent Publishing Platform, 2016

Course Assignments:

Attendance:

Class attendance and participation is required because the class is designed as a shared learning experience and because essential information not in the textbook will be discussed in class. You have to notify the instructor in advance of your absence. If you fail to attend class on a regular basis, your final course grade will be lowered. Likewise, you should arrive to class on time. Tardiness is disruptive and disrespectful to me and to your classmates. Please make every effort to arrive punctually.

Quizzes:

There will be five quizzes in total. Short, in-class quizzes will test your comprehension of course materials. You are supposed to make adequate preparation before each quiz. You are not allowed to consult your classmates or read your textbook or handout during the quizzes. You should be well-prepared before the class.

Homework:

There will be weekly homework assignments. Each assignment will be graded. Missing questions and answers without work do not earn credit. The due date for each homework assignment will be announced with the assignment. Late homework submission won't be accepted. You may work together and discuss homework. You may also ask your instructor for a hint.

Exams:



There will be two midterm exams and one final exam during the course. In the exams, you are responsible to explain theoretical concepts, answer problem questions related to theoretical concepts, make graphical representations, solve short numerical exercises. The exams will be close-book. Also, you are not allowed to communicate with your classmates. Students are required to take all exams, and there are NO MAKE-UP EXAMS.

Course Assessment:

| | |
|-----------------|-------------|
| Attendance | 10% |
| Quizzes | 10% |
| Homework | 10% |
| Midterm Exams 1 | 15% |
| Midterm Exams 2 | 20% |
| Final Exam | 35% |
| Total | 100% |

Grading Scale:

A+: 98%-100%

A: 93%-97%

A-: 90%-92%

B+: 88%-89%

B: 83%-87%

B-: 80%-82%

C+: 78%-79%

C: 73%-77%

C-: 70%-72%

D+: 68%-69%

D: 63%-67%

D-: 60%-62%

F: Below 60%

Course Schedule:

| Week | Topics | Assignments |
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| 1 | <ul style="list-style-type: none">• Introduction to The Course• Properties of real numbers<ul style="list-style-type: none">The Real Number SystemAlgebraic StructureOrder StructureBounds (Upper Bounds, Lower Bounds, Maximum, Minimum)Sups and Infs (Least Upper Bound/ Supremum, Greatest Lower Bound/ Infimum)The Archimedean PropertyInductive Property of \mathbb{N}The Rational Numbers Are DenseThe Metric Structure of \mathbb{R} (Absolute Value, Distance)• Sequences<ul style="list-style-type: none">SequencesCountable SetsConvergence and DivergenceBoundedness Properties of LimitsAlgebra of LimitsOrder Properties of Limits | <ul style="list-style-type: none">• Quiz 1• Homework 1 |
| 2 | <ul style="list-style-type: none">• Sequences<ul style="list-style-type: none">SubsequencesCauchy Convergence CriterionUpper and Lower Limits• Sets of Real Numbers<ul style="list-style-type: none">Points and setsElementary TopologyCompactness Arguments (Bolzano-Weierstrass Property, | <ul style="list-style-type: none">• Quiz 2• Homework 2• Quiz 3• Midterm Exam 1 |



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| | <p>Cantor's Intersection Property, Cousin's Property, Heine-Borel Property, Compact Sets)</p> <p>Countable Sets</p> <ul style="list-style-type: none">• Continuous Functions <p>Introduction to Limits</p> <p>Properties of Limits</p> <p>Limits Superior and Inferior</p> <p>Continuity</p> <p>Properties of Continuous Functions</p> <p>Uniform Continuity</p> <p>Extremal Properties</p> <p>Darboux Property</p> | |
| 3 | <ul style="list-style-type: none">• Continuous Functions <p>Points of Discontinuity</p> <p>Types of Discontinuity</p> <p>Monotonic Functions</p> <ul style="list-style-type: none">• The integral <p>Properties of the Integral</p> <p>Cauchy's First and Second Method</p> <p>The Riemann Integral and Its Properties</p> <p>The Improper Riemann Integral</p> <p>The Fundamental Theorem of Calculus</p> <ul style="list-style-type: none">• Sequences and Series of Functions <p>Pointwise limits and Uniform limits</p> <p>Uniform Convergence and Continuity</p> <p>Uniform Convergence and the Integral (Sequences of Continuous Functions, Sequences of Riemann Integrable Functions, Sequences of Improper Integrals)</p> <p>Uniform Convergence and Derivatives</p> <p>Pompeiu's Function</p> | <ul style="list-style-type: none">• Quiz 4• Homework 3• Midterm Exam 2• Homework 4 |



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|---|---|--|
| 4 | <ul style="list-style-type: none"> • Power Series <ul style="list-style-type: none"> Power Series: Convergence Uniform Convergence Functions Represented by Power Series The Taylor Series Products of Power Series Trigonometric Series (Uniform Convergence of Trigonometric Series, Fourier Series, Convergence of Fourier Series, Weierstrass Approximation Theorem) • The Euclidean Spaces R^n <ul style="list-style-type: none"> The Algebraic Structure of R^n The Metric Structure of R^n Elementary Topology of R^n Sequences in R^n Functions and Mappings Limits of Functions from $R^n \rightarrow R^m$ Continuity of Functions from $R^n \rightarrow R^m$ Compact sets in R^n • Review for Final Exam | <ul style="list-style-type: none"> • Quiz 5 • Homework 5 • Final Exam |
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Academic Integrity:

Students are encouraged to study together, and to discuss lecture topics with one another, but all other work should be completed independently.

Students are expected to adhere to the standards of academic honesty and integrity that are described in the Shanghai Normal University's *Academic Conduct Code*. Any work suspected of violating the standards of the *Academic Conduct Code* will be reported to the Dean's Office. Penalties for violating the *Academic Conduct Code* may include dismissal from the program. All students have an individual responsibility to know and understand the provisions of the *Academic Conduct Code*.



Special Needs or Assistance:

Please contact the Administrative Office immediately if you have a learning disability, a medical issue, or any other type of problem that prevents professors from seeing you have learned the course material. Our goal is to help you learn, not to penalize you for issues which mask your learning.